CSE 152 Section 2

Recap: Geometric Image Formation

April 08, 2019

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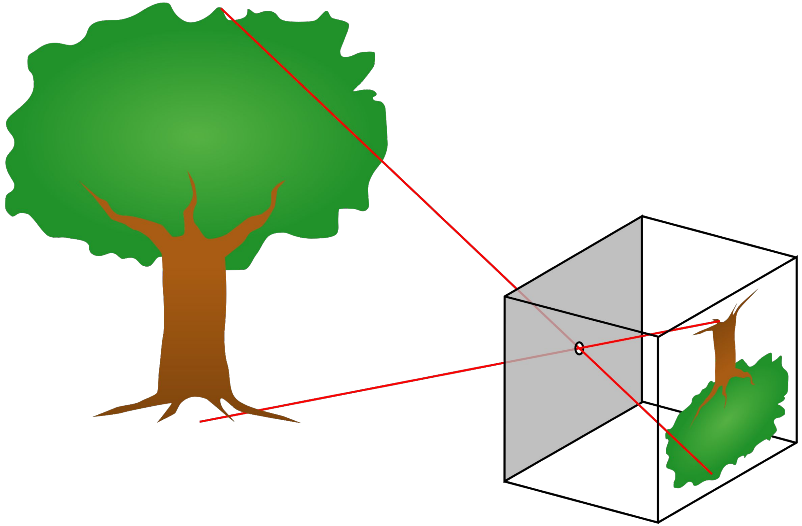
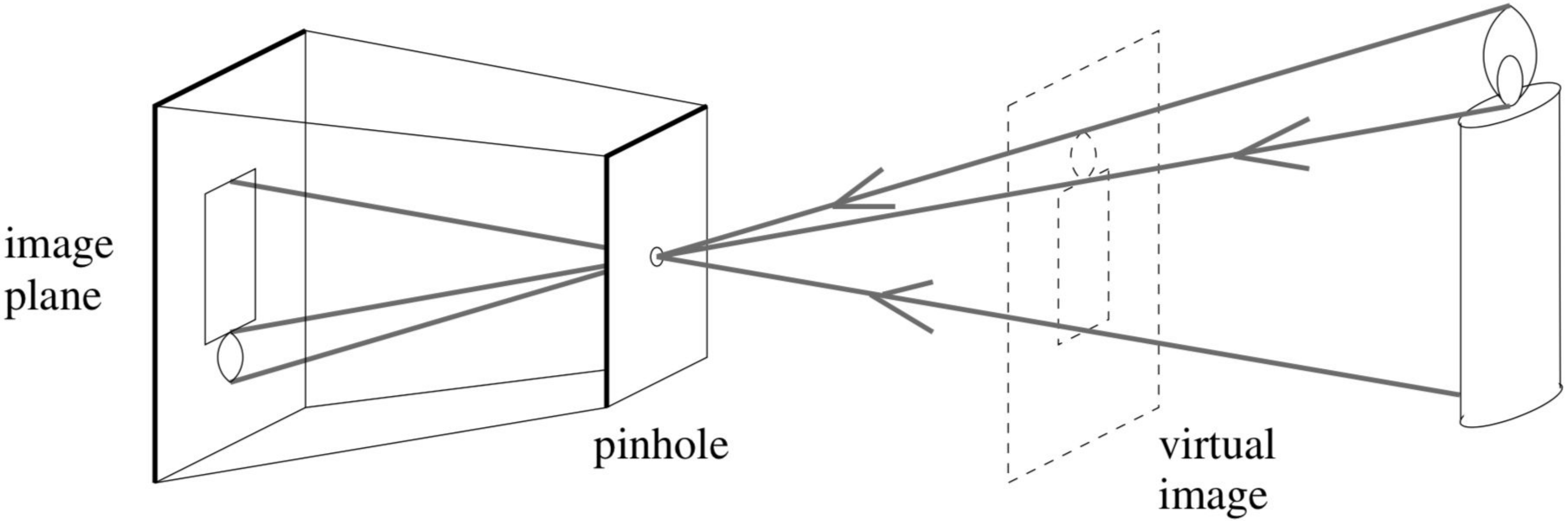
Pinhole Perspective Projection

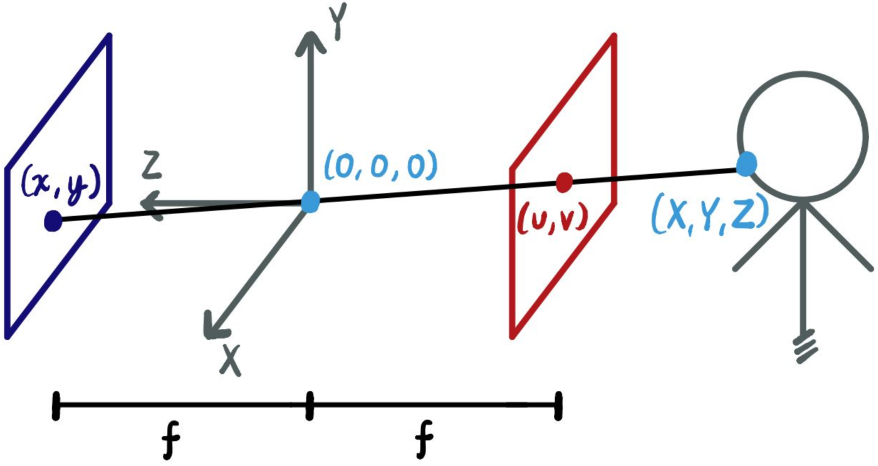
Light ray bounces off something in world, passes through pinhole, gets recorded on back wall of camera.



Virtual Image Plane



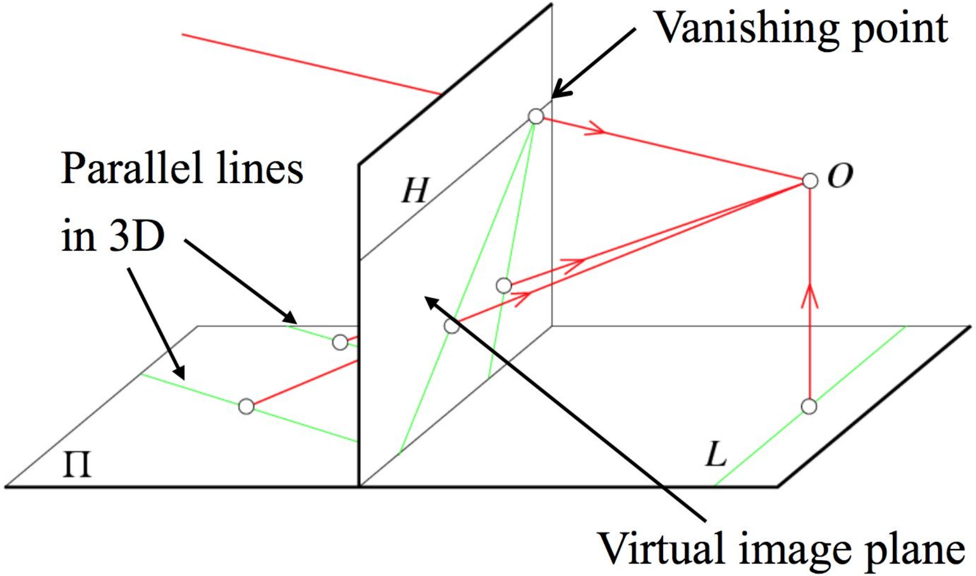
Relevant Equations



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \end{bmatrix} \qquad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -fX/Z \\ -fY/Z \end{bmatrix}$$

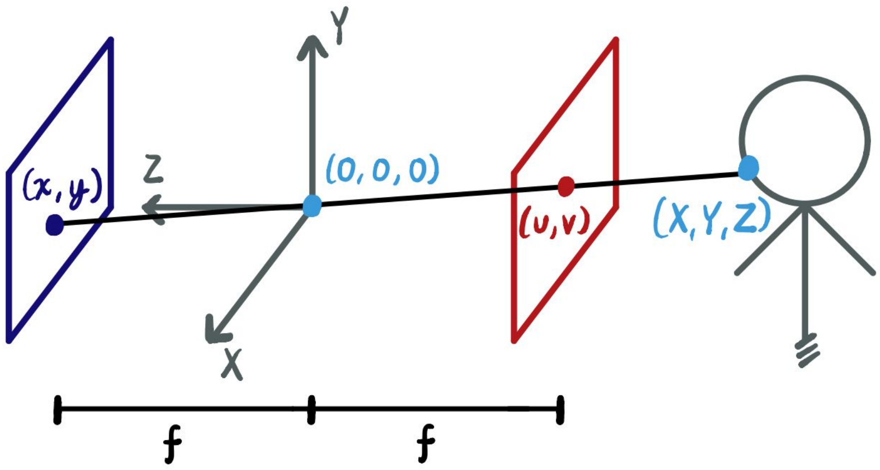
Vanishing Points

- Projected **point at infinity** (point to which line converges in projective space)



Homogeneous Coordinates

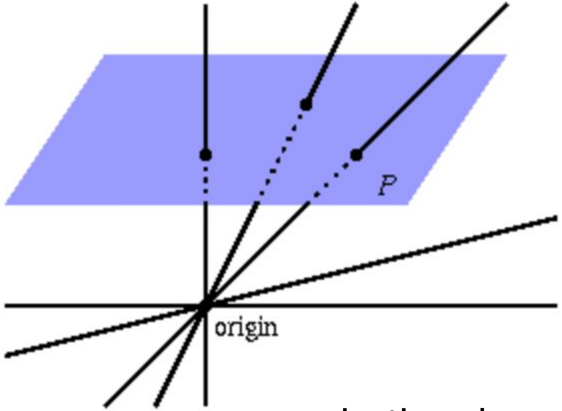
- Make translation and perspective projection “linear”



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} \qquad \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} -fX \\ -fY \\ Z \end{bmatrix}$$

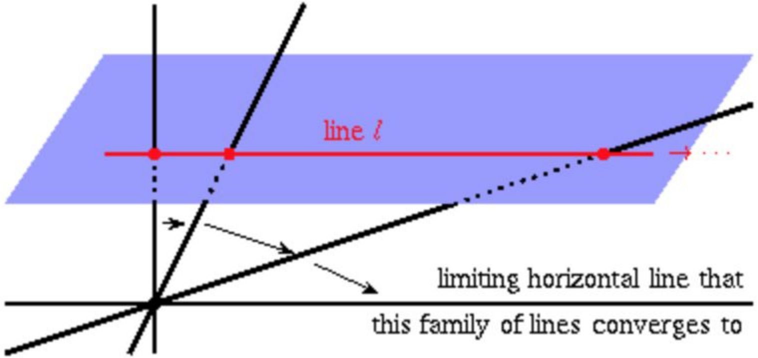
Projective Geometry

- Geometry in **projective space**
  - Euclidean space + points at infinity
  - Allow transformations between Euclidean points and points at infinity



projective plane

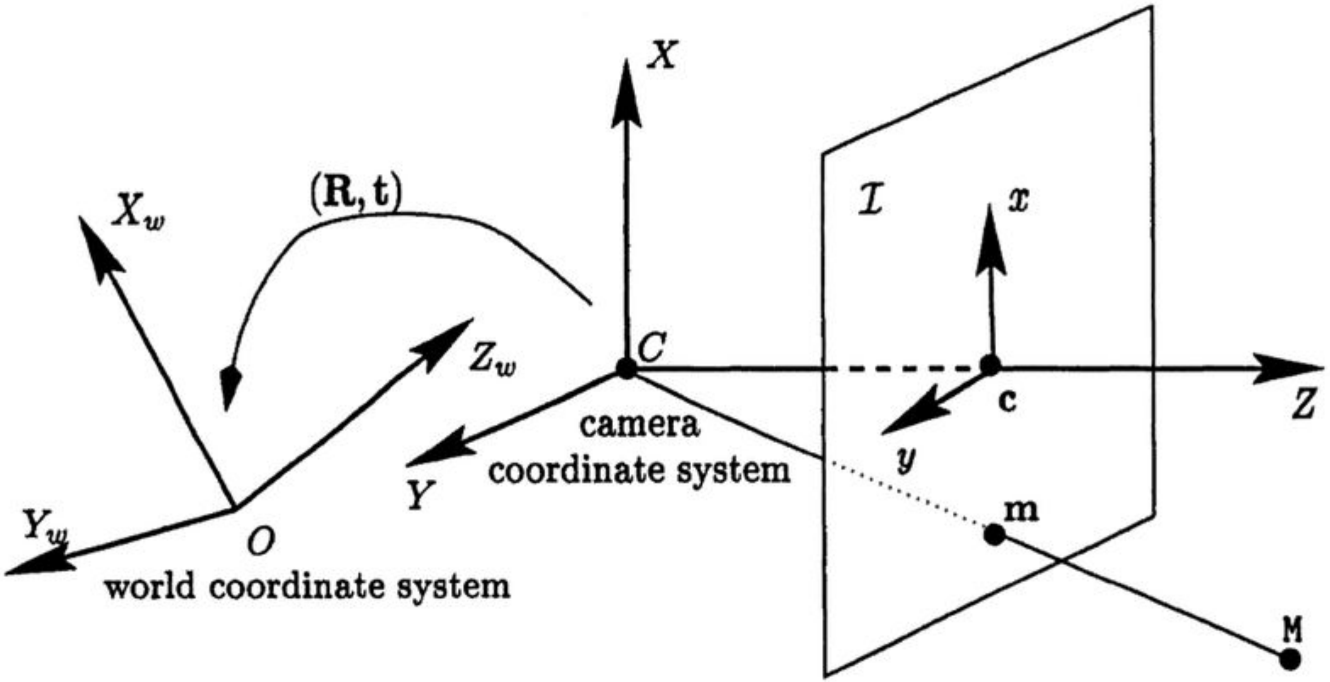
`Euclidean plane + points at infinity`



projective line

`Euclidean line + point at infinity`

The World Frame



3D Rotation

- 3D rotation matrix ∈ SO(3)
  - 3x3 orthogonal matrix → transpose is inverse
  - If you consider points as fixed and frame as changing, rows are new axes in current frame

$$
{}^{B}_{A}R = \begin{bmatrix} — & {}^{A}i_B & — \\ — & {}^{A}j_B & — \\ — & {}^{A}k_B & — \end{bmatrix}
$$

$$
{}^{B}_{A}R : \text{rotation matrix, transforms points in frame } A \text{ to points in frame } B
$$

$$
{}^{A}i_B,\ {}^{A}j_B,\ {}^{A}k_B : \text{frame } B \text{ axes in coordinate system } A
$$

Rigid Transformation

- Rotation, then translation
- Preserves distances between pairs of points

$$^{B}P = {}^{B}_{A}R\,{}^{A}P + {}^{B}O_A$$

frame A coordinates → frame B coordinates

$^{A}P,\ ^{B}P$: point in frame A, point in frame B

${}^{B}_{A}R$: rotation matrix (frame A coords $\rightarrow$ frame B coords)

$^{B}O_A$: frame A origin in frame B coordinates

Full Camera Projection Process

$$\underbrace{\mathbf{p}}_{\text{image point}} = \underbrace{\begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsic matrix}} \underbrace{\left[\begin{array}{ccc|c} - & - & - & - \\ - & \mathbf{R} & - & \mathbf{t} \\ - & - & - & - \end{array}\right]}_{\text{extrinsic matrix}} \underbrace{\mathbf{P}}_{\text{world point}}$$

homogeneous 3x1 point in image coords

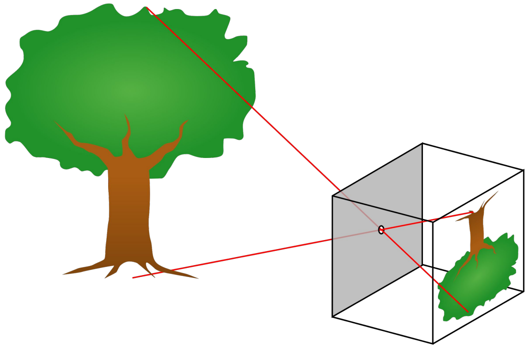
inhomogeneous* 3x1 point in camera coords

homogeneous 4x1 point in world coords

- **Extrinsic matrix**: world coords `(rigid transf.)` → camera coords
- **Intrinsic matrix**: camera coords `(projection)` → image coords

Section Takeaways

- How to:
  - perform perspective projection
  - compute locations of vanishing points
- Intuition about:
  - homogeneous coordinates and projective geometry
  - 3D rotations and rigid transformations
  - full camera projection process



Q & A