



# CS 61A

## DISCUSSION NINE

November 10, 2016



# TOPICS FOR TODAY

Iterators, generators, streams (, ...turtle graphics demo !?)

## Overview

- ▶ Iterators are objects that sweep over a collection of items (in a specific order / step size). This happens via repeated application of the `next` method, which is necessarily defined on all iterators.
- ▶ Streams are lazily evaluated linked lists. Elements in the list aren't computed until you specifically ask for them, which means you can represent infinite sequences as streams.

# ANNOUNCEMENTS

- ▶ No in-person lab on Thanksgiving week (instead, we'll have a “long, optional” one to be completed on your own)

# ITERATORS

# ITERATION (REVIEW)

**Iterators** step through a collection, item by item, via `next`

**Iterables** “are” the collection, and provide **iterators** via `iter`

If you have an **iterable**, you can get an **iterator** over it by calling `iter`. Then you can observe all of its elements by repeatedly calling `next` on the **iterator**.

Be warned: iterators are single-use only! (Once an iterator has gone through all the elements, it’s done; calling `next` on it will give `StopIteration` errors forever.) To get a fresh iterator, one would have to call `iter` on the iterable again.

- Note that depending on the implementation, the second iterator might be finished anyway (e.g. if every iterator modifies the same state variables)

# ITERATION (REVIEW)

`iter(iterable)` → iterator

`next(iterator)` → value, or a `StopIteration` error

# ITERATION (REVIEW)

Lots of **built-in** functions take or produce iterators! Be aware of these.

- ▶ `map(function, iterable)`
  - ▷ Returns an **iterator** over mapped elements in the iterable
- ▶ `filter(function, iterable)`
  - ▷ Returns an **iterator** over filtered elements from the iterable
- ▶ `zip(*iterables)`
  - ▷ Returns an **iterator** over aggregations of elements from each of the iterables

# ITERATION (REVIEW)

To create an iterable, you could write a class that implements `__iter__` as a generator function.

```
class Primes:
    def __init__(self, n):
        self.n = n # upper limit
    def __iter__(self):
        P = [True for i in range(2, self.n + 1)]
        for i in range(2, self.n + 1):
            if not P[i - 2]: continue
            yield i
            if i > sqrt(self.n): continue
            for j in [i*i + k*i for k in range((self.n - i*i)//i + 1)]:
                P[j - 2] = False
```

```
list(Primes(30)) → [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
```

# ITERATION (REVIEW)

Behind the scenes, for-loops really just create iterators using `iter` and then call `next` a bunch of times on the iterator.

```
for x in <expr>:
    <do stuff>
– is equivalent to –
iterator = iter(<expr>)
try:
    while True:
        n = next(iterator)
        <do stuff>
except StopIteration:
    pass
```

# ITERATORS E1

```
>>> t = iter([0, 1, 2]) # t is now an iterator  
>>> u = iter(t) # does it error..?
```

---

What happens when you call `iter` on an iterator?

# ITERATORS E1

```
>>> t = iter([0, 1, 2]) # t is now an iterator
>>> u = iter(t) # does it error..?
```

---

What happens when you call `iter` on an iterator?

You get the same iterator back. So... technically speaking, iterators also implement the iterable interface! (Unless they're user-defined iterators that didn't implement the `__iter__` method, but you don't need to worry about that this semester...)

```
>>> t == u
True
>>> t == iter(t)
True
```

# ITERATORS E2

Let's implement the `list` function. Here we have the function specification:

```
def list(iterable):  
    """Creates a list.  
  
    >>> list(range(4))  
    [0, 1, 2, 3]  
    >>> list(iter(range(4)))  
    [0, 1, 2, 3]  
    """  
  
    # YOUR CODE HERE
```

# ITERATORS E2

Let's implement the `list` function. Here we have the function specification:

```
def list(iterable):  
    iterator = iter(iterable)  
    result = []  
    try:  
        while True:  
            result.append(next(iterator))  
    except StopIteration:  
        return result
```

# GENERATORS

# GENERATORS (REVIEW)

**Generator functions** are functions containing `yield` statements.

- ▶ These functions return generators when called.

**Generators** are **iterators** obtained by calling a generator function.

- ▶ Every time you call `next` on a generator, it goes through the generator function body until it hits a `yield` – at which point it yields the specified value. The state of the function with respect to this generator is saved, so whenever `next` is called again on the generator, execution of the function body will continue from where it left off.

To create a fresh iterator, just call the generator function again.

# GENERATORS (REVIEW)

Here we have a function that returns an iterator over the natural numbers:

```
def gen_naturals():
    curr = 0
    while True:
        yield curr
        curr += 1

>>> gen = gen_naturals()
>>> gen
<generator object gen at ...>
>>> next(gen)
0
>>> next(gen)
1
```

# GENERATORS (REVIEW)

``yield from`` is like `yield`, except it yields **all values** from an **iterable**.

```
yield from <expr>
```

– *is equivalent to* –

```
for x in <expr>:
```

```
    yield x
```

# GENERATORS (REVIEW)

```
square = lambda x: x * x
def many_squares(s):
    for x in s:
        yield square(x)
    yield from [square(x) for x in s]
    yield from map(square, s)
```

```
>>> list(many_squares([1, 2, 3]))
[1, 4, 9, 1, 4, 9, 1, 4, 9]
```

# GENERATORS E1

```
>>> def gen_y():  
...     yield (7, 8, 9)  
...  
>>> def gen_yf():  
...     yield from (7, 8, 9)  
...  
>>> next(gen_y())  
  
>>> next(gen_yf())
```

# GENERATORS E1

```
>>> def gen_y():
...     yield (7, 8, 9)
...
>>> def gen_yf():
...     yield from (7, 8, 9)
...
>>> next(gen_y())
(7, 8, 9)

>>> next(gen_yf())
7
```

## GENERATORS E2

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup)
```

---

What would Python print?

## GENERATORS E2

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup)
```

---

What would Python print?

Trick question, `countup` is just a generator function (NOT a generator; NOT an iterable)

(`TypeError: 'function' object is not iterable`)

## GENERATORS E3

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup())
```

---

What would Python print?

## GENERATORS E3

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup())
```

---

What would Python print?

**[1, 2, 3]**

Return results in a `StopIteration` (incl. the implicit return at the end of the function). The value returned is not actually returned by the generator function.

## GENERATORS E4

```
>>> def myst(iterable, n):
...     if n <= 0: return
...     iterator = iter(s0)
...     yield next(iterator)
...     if n % 2:
...         yield from myst(iterable, n - 1)
...     else:
...         yield from myst(iterator, n - 1)
...
>>> gen = lambda: (yield from range(7))
>>> [e for e in myst(gen(), 3)]
```

---

What would Python print?

## GENERATORS E4

```
>>> def myst(iterable, n):
...     if n <= 0: return
...     iterator = iter(s0)
...     yield next(iterator)
...     if n % 2:
...         yield from myst(iterable, n - 1)
...     else:
...         yield from myst(iterator, n - 1)
...
>>> gen = lambda: (yield from range(7))
>>> [e for e in myst(gen(), 3)]
```

---

[0, 1, 2]. Note that ``yield from myst(iterable, n - 1)`` is equivalent to ``yield from myst(iterator, n - 1)`` in this case. (Why?)

## GENERATORS E5

```
>>> def myst(iterable, n):
...     if n <= 0: return
...     iterator = iter(s0)
...     yield next(iterator)
...     if n % 2:
...         yield from myst(iterable, n - 1)
...     else:
...         yield from myst(iterator, n - 1)
...
>>> [e for e in myst(range(7), 3)]
```

---

Now what would Python print?

(This is exactly the same code as last time, except we're calling `myst` on a different iterable.)

## GENERATORS E5

```
>>> def myst(iterable, n):
...     if n <= 0: return
...     iterator = iter(s0)
...     yield next(iterator)
...     if n % 2:
...         yield from myst(iterable, n - 1)
...     else:
...         yield from myst(iterator, n - 1)
...
>>> [e for e in myst(range(7), 3)]
```

---

[0, 0, 1]. Why? Because `iter` is returning a fresh iterator every time! (In the last case, it wasn't – because the iterable that `iter` was being called on was an iterator. *Note, again: iterators are technically iterables because you can call `iter` on them.*)

## GENERATORS E6

```
>>> def gen_e6(n):  
...     yield n / (n - 1)  
...     yield from gen_e6(n - 1)
```

```
...  
>>> e6 = gen_e6(3)  
>>> next(e6)
```

```
>>> next(e6)
```

```
>>> next(e6)
```

```
>>> next(e6)
```

## GENERATORS E6

```
>>> def gen_e6(n):  
...     yield n / (n - 1)  
...     yield from gen_e6(n - 1)  
...  
>>> e6 = gen_e6(3)  
>>> next(e6)  
1.5  
  
>>> next(e6)  
2.0  
  
>>> next(e6)  
ZeroDivisionError: division by zero  
  
>>> next(e6)  
StopIteration
```

## GENERATORS 1.5.1

Write a generator that combines identically-indexed elements of two iterators using a given combiner function. When either iterator runs out of elements, the generator as a whole should also run out of elements.

```
def combiner(iterator1, iterator2, combiner):  
    # YOUR-CODE-HERE
```

```
>>> from operator import add  
>>> evens = combiner(gen_naturals(), gen_naturals(), add)  
>>> next(evens)  
0  
>>> next(evens)  
2  
>>> next(evens)  
4
```

## GENERATORS 1.5.1/2

```
def combiner(iterator1, iterator2, combiner):  
    while True:  
        yield combiner(next(iterator1), next(iterator2))
```

WWPP?

```
>>> nats = gen_naturals()  
>>> doubled_nats = combiner(nats, nats, add)  
>>> next(doubled_nats)  
  
>>> next(doubled_nats)
```

## GENERATORS 1.5.1/2

```
def combiner(iterator1, iterator2, combiner):  
    while True:  
        yield combiner(next(iterator1), next(iterator2))
```

WWPP?

```
>>> nats = gen_naturals()  
>>> doubled_nats = combiner(nats, nats, add)  
>>> next(doubled_nats)  
1  
>>> next(doubled_nats)  
5
```

## GENERATORS 1.5.3

Write a generator function that goes through all subsets of the positive integers from 1 to  $n$ . Each call to this generator's `next` method will return a list of subsets of the set  $[1, 2, \dots, n]$ , where  $n$  is the number of times `next` was previously called.

```
def generate_subsets():  
    """  
    >>> subsets = generate_subsets()  
    >>> for _ in range(3):  
    ...     print(next(subsets))  
    ...  
    [[]]  
    [[], [1]]  
    [[], [1], [2], [1, 2]]  
    """  
    # YOUR-CODE-HERE
```

## GENERATORS 1.5.3

```
def generate_subsets():  
    # YOUR-CODE-HERE
```

Thought process:

- ▶ Uh...
- ▶ Okay, well it's a generator function so we're going to have to yield stuff
- ▶ Looking at the doctest, it seems as if we always want the positive integers to be in order. We're just splitting them up
  - ▷  `[[]]`
  - ▷  `[[], [1]]`
  - ▷  `[[], [1], [2], [1, 2]]`
- ▶ What do you notice? Each successive yield is just *everything from before*, and also *everything from before* with the latest value of `n` tacked onto the end

## GENERATORS 1.5.3

```
def generate_subsets():  
    # YOUR-CODE-HERE
```

In other words,

- ▶  $n = 0$ :  
[[]]
- ▶  $n = 1$ :  
[[], [1]], which is really just  
[[] AND [] + [1]]
- ▶  $n = 2$ :  
[[], [1], [2], [1, 2]], which is really just  
[[], [1] AND [] + [2], [1] + [2]]
- ▶  $n = 3$ :  
[[], [1], [2], [1, 2], [3], [1, 3], [2, 3], [1, 2, 3]], or  
[... AND [] + [3], [1] + [3], [2] + [3], [1, 2] + [3]]

## GENERATORS 1.5.3

```
def generate_subsets():  
    n, subsets = 1, [[]]  
    while True:  
        yield subsets  
        subsets += [s + [n] for s in subsets]  
        n += 1
```

A large, solid blue diagonal shape that starts from the top right and extends towards the bottom left, creating a split background of white and blue.

**STREAMS**

# STREAMS

Streams are linked lists that are evaluated lazily:

- ▶ The rest won't be computed until we ask for it
- ▶ After we ask for it, the result will be **remembered**

Scheme stream interface:

- ▶ `car` gives us the first element of the stream
- ▶ `nil` is the empty stream
- ▶ `cons-stream`: like `cons`, except the rest isn't evaluated at first
- ▶ `cdr-stream`: like `cdr`, but tells the stream **to actually compute the rest** if it hasn't already. Note: don't use `cdr` with streams!

# STREAMS

Non-required material, but you'll probably run into it so

## Scheme promises (force/delay)

- ▶ How lazily evaluated expressions are actually implemented in Scheme
- ▶ [Spec description](#)
- ▶ **Promise:** a delayed expression
  - ▷ Can be “forced” (already evaluated and now cached)
  - ▷ or “not forced” (not evaluated yet)
- ▶ If you display a stream in the interpreter, you'll see this

```
scm> (cons-stream 1 2)
(1 . #[promise (not forced)])
```

# STREAMS

What is the advantage of using a stream over a linked list?

# STREAMS

What is the advantage of using a stream over a linked list?

Elements won't be evaluated unnecessarily if they are never used... meaning efficient space usage! Also, streams allow for the representation of infinite-length sequences.

*On streams versus iterators:*

Every time you call `next` on an iterator, it changes. The nice thing about streams is that they never change.

# STREAMS E1

We attempt to define an infinite sequence of natural numbers.

```
(define (naturals n)
  (cons n (naturals (+ n 1))))
```

---

```
(define nat (naturals 0))
```

What happens?

# STREAMS E1

We attempt to define an infinite sequence of natural numbers.

```
(define (naturals n)
  (cons n (naturals (+ n 1))))
```

---

```
(define nat (naturals 0))
```

Error: maximum recursion depth exceeded

# STREAMS E1

That didn't work, so we turn to streams instead.

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

---

```
(define nat (naturals 0))
```

```
(car nat)
```

```
(car (cdr-stream nat))
```

```
(car (cdr-stream (cdr-stream nat)))
```

# STREAMS E1

That didn't work, so we turn to streams instead.

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

---

```
(define nat (naturals 0)) → nat
```

```
(car nat) → 0
```

```
(car (cdr-stream nat)) → 1
```

```
(car (cdr-stream (cdr-stream nat))) → 2
```

# STREAMS E1

That didn't work, so we turn to streams instead.

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

---

```
(define nat (naturals 0)) → nat
```

```
(cdr nat)
```

```
(cdr-stream nat)
```

```
(cdr nat)
```

# STREAMS E1

That didn't work, so we turn to streams instead.

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

---

```
(define nat (naturals 0)) → nat
```

```
(cdr nat) → #[promise (not forced)]
```

```
(cdr-stream nat) → (1 . #[promise (not forced)])
```

```
(cdr nat) → #[promise (forced)]
```

## STREAMS 2.1.1 - WWSD?

```
scm> (define (has-even? s)
      (cond ((null? s) False)
            ((even? (car s)) True)
            (else (has-even? (cdr-stream s)))))
```

has-even?

```
scm> (define ones (cons-stream 1 ones))
```

ones

```
scm> (define twos (cons-stream 2 twos))
```

twos

```
scm> ones
```

```
scm> (cdr-stream ones)
```

```
scm> (has-even? ones)
```

```
scm> (has-even? twos)
```

## STREAMS 2.1.1 - WWSD?

```
scm> (define (has-even? s)
      (cond ((null? s) False)
            ((even? (car s)) True)
            (else (has-even? (cdr-stream s)))))
```

has-even?

```
scm> (define ones (cons-stream 1 ones))
```

ones

```
scm> (define twos (cons-stream 2 twos))
```

twos

```
scm> ones
```

```
(1 . #[promise (not forced)])
```

```
scm> (cdr-stream ones)
```

```
(1 . #[promise (forced)])
```

```
scm> (has-even? ones)
```

Runs forever

```
scm> (has-even? twos)
```

True

## STREAMS 2.1.2

Implement `map-stream`, which maps a function `f` to a stream `s`.

```
(define (map-stream f s)
  ; YOUR-CODE-HERE
)
```

```
scm> (define evens (map-stream (lambda (x) (* x 2)) nat))
```

```
evens
```

```
scm> (car (cdr-stream evens))
```

```
2
```

```
scm> (car (cdr-stream (cdr-stream evens)))
```

```
4
```

## STREAMS 2.1.2

```
(define (map-stream f s)
  ; YOUR-CODE-HERE
)
```

Approach (what do we know?):

- ▶ We need to create a new stream
- ▶ We need to apply the function to every element
- ▶ We need to not think about the enormity of applying a function to every element in an infinite sequence, and instead consider the problem inductively

## STREAMS 2.1.2

```
(define (map-stream f s)
  ; YOUR-CODE-HERE
)
```

Approach (what do we know?):

- ▶ We need to create a new stream
  - ▷ So we'll use `cons-stream`
- ▶ We need to apply the function to every element
  - ▷ So we'll do `(f (car s))`
- ▶ We need to not think about the enormity of applying a function to every element in an infinite sequence, and instead consider the problem inductively
  - ▷ So we'll recurse, creating the rest of the stream via `(map-stream f (cdr-stream s))`. Our base case will be an empty stream

## STREAMS 2.1.2

```
(define (map-stream f s)
  (if (null? s) nil
      (cons-stream (f (car s))
                    (map-stream f (cdr-stream s))))
)
```

## STREAMS 2.1.3

Consider the following two implementations of `filter-stream`. One is correct and one is not. Which is the incorrect one, and why?

```
(define (filter-stream f s)
  (if (null? s) nil
      (let ((rest (filter-stream f (cdr-stream s))))
        (if (f (car s))
            (cons-stream (car s) rest)
            rest))))
```

```
(define (filter-stream f s)
  (cond ((null? s) nil)
        ((f (car s)) (cons-stream (car s) (filter-stream f (cdr-stream s))))
        (else (filter-stream f (cdr-stream s)))))
```

## STREAMS 2.1.3

The first one is incorrect, because it recursively creates the rest of the stream either until it hits the end of `s` or until it hits the maximum recursion depth. We want lazy evaluation, so we shouldn't be constructing the entire stream at once!

```
(define (filter-stream f s)
  (if (null? s) nil
      (let ((rest (filter-stream f (cdr-stream s))))
        (if (f (car s))
            (cons-stream (car s) rest)
            rest))))
```

```
(define (filter-stream f s)
  (cond ((null? s) nil)
        ((f (car s)) (cons-stream (car s) (filter-stream f (cdr-stream s))))
        (else (filter-stream f (cdr-stream s)))))
```

## STREAMS 2.1.4

Write a function, `range-stream`, that takes in two integers `start` and `end` and returns the same thing that `range(start, end)` would... but as a stream.

```
(define (range-stream start end)
  ; YOUR-CODE-HERE
)
```

```
scm> (define rs (range-stream 1 3))
rs
scm> (car rs)
1
scm> (car (cdr-stream rs))
2
scm> (cdr-stream (cdr-stream rs))
()
```

## STREAMS 2.1.4

```
(define (range-stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ We have to create a stream where the first element is `start`
- ▶ The “rest” of the stream should be another stream whose first element is `start + 1`
- ▶ ^ Sounds like recursion to me. We can just make the rest of the stream be the return value of a `range-stream` call
- ▶ The base case, then, will be when `start >= end`. In this case, there’s really no range to speak of – so we return an empty stream

## STREAMS 2.1.4

```
(define (range-stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ We have to create a stream where the first element is `start`
  - ▷ `(cons-stream start ...)`
- ▶ The “rest” of the stream should be another stream whose first element is `start + 1`
  - ▷ `(cons-stream (+ start 1) ...)`
- ▶ ^ Sounds like recursion to me. We can just make the rest of the stream be the return value of a `range-stream` call
  - ▷ `(cons-stream start (range-stream (+ start 1) end))`
- ▶ The base case, then, will be when `start >= end`. In this case, there’s really no range to speak of – so we return an empty stream
  - ▷ `(if (>= start end) nil ...)`

## STREAMS 2.1.4

```
(define (range-stream start end)
  (if (>= start end) nil
      (cons-stream start (range-stream (+ start 1) end))))
)
```

## STREAMS 2.1.5

Write a function, `slice`, that returns a **list** containing the elements of `stream` from index `start` to index `end - 1`. If you run out of elements in `stream`, just cut the list short.

```
(define (slice stream start end)
  ; YOUR-CODE-HERE
)
```

## STREAMS 2.1.5

```
(define (slice stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ We're returning a list! Back to `cons` and `list` and `append...` and no lazy evaluation. We need everything *now*
- ▶ Let's just recurse our way through `stream` decrementing `start` until it hits 0, and *then* start adding elements to our output list
- ▶ How will we know when to stop adding stuff? We want `end - start` elements overall... oh, we should also decrement `end` in that last step. Then, when `start` reaches 0, we'll know we want to add `end - 0` elements
- ▶ So we continue, decrementing `end` and adding elements to our list, until `end` is equal to 0. At this point, we'll be done

## STREAMS 2.1.5

```
(define (slice stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process, continued:

- ▶ We can use `cons` since it fits: we have an element and a “rest” list to use in the `(cons element rest)` formula
  - ▷ Specifically, we have `(cons (car stream) <recursive call>)`
- ▶ We should also check if `stream` is empty, since it's not guaranteed that our indices are in-range

## STREAMS 2.1.5

Bringing it all together, we have

```
(define (slice stream start end)
  (cond ((or (= end 0) (null? stream)) nil)
        ((> start 0) (slice (cdr-stream stream)
                              (- start 1)
                              (- end 1)))
        (else (cons (car stream)
                     (slice (cdr-stream stream)
                             0
                             (- end 1))))))
)
```

## STREAMS 2.1.6

Let's combine infinite-length streams using `zip-with`, which accepts a function `f` and two streams (`xs` and `ys`) and returns a single stream containing every pair of corresponding elements combined with `f`.

If the elements in `xs` were `x1`, `x2`, `x3`, ... and the elements in `ys` were `y1`, `y2`, `y3`, ..., then `(zip-with f xs ys)` would return a stream with the elements `(f x1 y1)`, `(f x2 y2)`, `(f x3 y3)`, ...

```
(define (zip-with f xs ys)
  (if (or (null? xs) (null? ys)) nil
      (cons-stream (f (car xs) (car ys))
                    (zip-with f (cdr-stream xs) (cdr-stream ys))))
)
```

## STREAMS 2.1.6

As an example, we can create the even numbers by adding every natural number to itself (s.t. the output sequence is  $0 + 0, 1 + 1, 2 + 2, \dots$ ).

```
scm> (define evens (zip-with + (naturals 0) (naturals 0)))  
evens  
scm> (slice evens 0 10)  
(0 2 4 6 8 10 12 14 16 18)
```

## STREAMS 2.1.6

How would we define a stream containing the factorials of all numbers in order (starting at 0!)? **Use zip-with!**

```
(define factorials
  ; YOUR-CODE-HERE
)
```

```
scm> (slice factorials 0 10)
(1 1 2 6 24 120 720 5040 40320 362880)
```

## STREAMS 2.1.6

```
(define factorials
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ `zip-with`? What does that do again? Oh yeah...
- ▶ Factorials? What are those again? Oh yeah...
  - ▷  $0! = 1$
  - ▷  $1! = 1 * 0! = 1 * 1$
  - ▷  $2! = 2 * 1! = 2 * 1 * 1$
  - ▷  $3! = 3 * 2! = 3 * 2 * 1 * 1$
- ▶ Interesting. It seems like we'd want to combine elements with `*` (multiplication)
- ▶ Well, based on the recurrences above, we'd want to multiply every natural number with the previous factorial. But we're *computing* the factorials! This is going to be trippy...

## STREAMS 2.1.6

```
(define factorials
  (cons-stream 1 (zip-with * factorials (naturals 1)))
)
```

- ▶ `cons-stream` is kind of like a base case (it's the anchor; the first element of the output stream)
- ▶ Observing the sequence:
  - ▷ First element is 1 (obvious)
  - ▷ Second element is  $[\wedge \text{that element}] * 1 = 1 * 1 = 1$
  - ▷ Third element is  $[\wedge \text{that element}] * 2 = 1 * 2 = 2$
  - ▷ Fourth element is  $[\wedge \text{that element}] * 3 = 2 * 3 = 6$
  - ▷ Fifth element is  $[\wedge \text{that element}] * 4 = 6 * 4 = 24$
  - ▷ ...
- ▶ I always thought this was neat af

## STREAMS 2.1.6

Last one. How would we define a stream containing the Fibonacci numbers, starting with 0 and 1? **Use zip-with!**

```
(define fibs
  ; YOUR-CODE-HERE
)
```

```
scm> (slice fibs 0 10)
(0 1 1 2 3 5 8 13 21 34)
```

## STREAMS 2.1.6

```
(define fibs
  ; YOUR-CODE-HERE
)
```

Let's do this

- ▶ We've been computing Fibonacci numbers since day 0
  - ▷  $F(0) = 0$
  - ▷  $F(1) = 1$
  - ▷  $F(2) = F(0) + F(1)$
  - ▷  $F(3) = F(1) + F(2)$
  - ▷ ...
- ▶ Looks like we're gonna be combining with `+` (addition)
- ▶ Two "base cases", 0 and 1, and then we'll zip-with `+` a `fibs` and a staggered `fibs`

## STREAMS 2.1.6

```
(define fibs
  (cons-stream 0 (cons-stream 1
    (zip-with + fibs (cdr-stream fibs))))
)
```

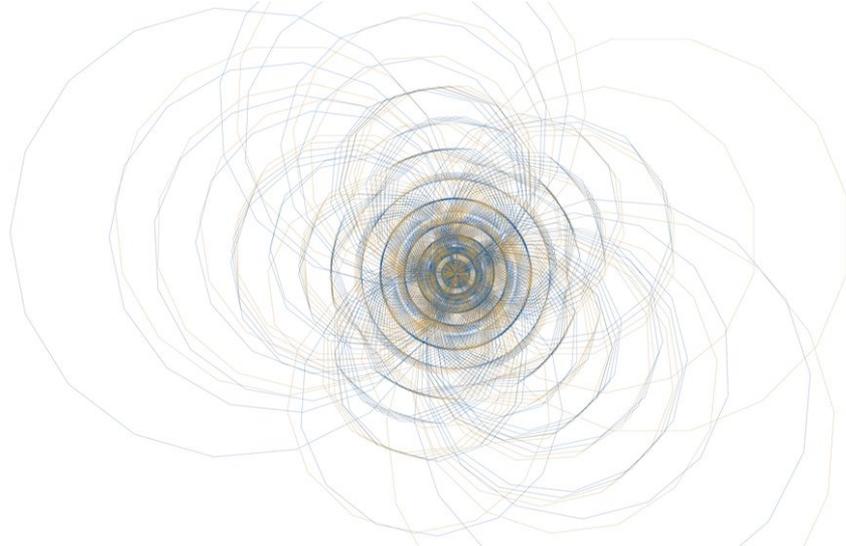
It's so beautiful...

**TURTLE**

# RECURSIVE ART CONTEST

It's optional... but it's also cool. Even if you don't enter the contest, try playing around with the turtle graphics system for a bit!

Note: you can probably sell a lot of things as a recursive process



# VECTOR GRAPHICS

Represent images using **geometric primitives** (points, lines, curves, shapes, polygons...) instead of **pixel values**. Based on **vectors** passing through control points, hence the name

As it happens, turtle graphics are vector-based!

# DEMO: IMAGE VECTORIZATION

Something you can do with turtle graphics:

- ▶ Take a photograph (or grab any image you like)
- ▶ Convert the image to SVG format (<https://www.vectorizer.io/>)
- ▶ Implement turtle functions for all primitives in the VG format you're using. This may only be path (a composite Bézier curve)!
- ▶ Convert the vector image into turtle code (using the primitives you just wrote) via a parser
- ▶ Let the turtle do its thing
- ▶ Profit

As I can show... **(edit: next time!)**

# DISCUSSION ATTENDANCE

<http://tiny.cc/5184>

# QUIZ 9