CSE 252B Lecture 05

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3D → 2D PROJECTION

$\underline{x} = \underline{P}\underline{X}$ point to point

$\underline{C}^* = \underline{P}\underline{Q}^*\underline{P}^T$ dual quadric to dual conic

$[\underline{l}]_x = \underline{P}\underline{L}\underline{P}^T$ line to line

2D → 3D BACK-PROJECTION

$\underline{L}^* = \underline{P}^T \overset{\text{(matrix form of cross product)}}{[\underline{x}]_x} \underline{P}$ point to dual line

$\underline{\pi} = \underline{P}^T\underline{l}$ line to plane

$\underline{Q}_{CONE} = \underline{P}^T \underset{\substack{\text{3×3 conic} \\ \text{(not camera center)}}}{\underline{C}} \underline{P}$ conic to cone (degenerate conic; $\underline{Q}_{CONE}\underset{\substack{\text{4×1 camera center} \\ \text{(vertex of cone)} \\ \text{(not conic)}}}{\underline{C}}$)

$\underline{P}$ MATRIX, LINEAR ESTIMATE

DIRECT LINEAR TRANSFORMATION (DLT) ALGORITHM

*do not use cross product, use left nullspace approach shown below

GIVEN: correspondences $\overset{\text{3×1 2D POINT}}{\underline{x}_i} \leftrightarrow \overset{\text{4×1 3D POINT}}{\underline{X}_i}$

FIND: a $\underline{P}$ s.t. $\underline{x}_i = \underline{P}\underline{X}_i \; \forall i$ (camera projection matrix)

(left nullspace)

$$[\underline{x}]^{\perp}\underline{x} = \underline{0}$$

$$\begin{bmatrix} \underline{l}_1^T \\ \underline{l}_2^T \end{bmatrix} \underline{x} = \underline{0}$$

for $\underline{l} = (a, b, c)^T$

$$\begin{bmatrix} \underline{l}_1^T \\ \underline{l}_2^T \end{bmatrix} \underline{P}\underline{X} = \underline{0}$$

for $\underline{P} = \begin{cases} \begin{bmatrix} \underline{p}^{1T} \\ \underline{p}^{2T} \\ \underline{p}^{3T} \end{bmatrix} & \text{where } \underline{p}^{iT} \text{ is } i^{th} \text{ row of } \underline{P} \text{ (SUPERSCRIPTS)} \\ [\underline{p}_1 \,|\, \underline{p}_2 \,|\, \underline{p}_3 \,|\, \underline{p}_4] & \text{where } \underline{p}_i \text{ is } i^{th} \text{ column of } \underline{P} \text{ (SUBSCRIPTS)} \\ \underset{3\times 4}{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}} & \end{cases}$

$$\begin{bmatrix} \underline{l}_1^T \\ \underline{l}_2^T \end{bmatrix} \begin{bmatrix} \underline{p}^{1T} \\ \underline{p}^{2T} \\ \underline{p}^{3T} \end{bmatrix} \underline{X} = \underline{0}$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} \underline{p}^{1T}\underline{X} \\ \underline{p}^{2T}\underline{X} \\ \underline{p}^{3T}\underline{X} \end{bmatrix} = \underline{0}$$

$$\begin{bmatrix} a_1\underline{p}^{1T}\underline{X} + b_1\underline{p}^{2T}\underline{X} + c_1\underline{p}^{3T}\underline{X} \\ a_2\underline{p}^{1T}\underline{X} + b_2\underline{p}^{2T}\underline{X} + c_2\underline{p}^{3T}\underline{X} \end{bmatrix} = \underline{0}$$

$$\underset{2\times 12}{\begin{bmatrix} a_1\underline{X}^T & b_1\underline{X}^T & c_1\underline{X}^T \\ a_2\underline{X}^T & b_2\underline{X}^T & c_2\underline{X}^T \end{bmatrix}} \underset{12\times 1}{\begin{bmatrix} \underline{p}^1 \\ \underline{p}^2 \\ \underline{p}^3 \end{bmatrix}} = \underline{0}$$

$$\begin{bmatrix} \mathbf{l}_1^T \otimes \mathbf{X}^T \\ \mathbf{l}_2^T \otimes \mathbf{X}^T \end{bmatrix} \mathbf{p} = \mathbf{0}$$

(Kronecker product)

for $\mathbf{p} = \text{vec}(\mathbf{P}^T)$
(VECTORIZE THE MATRIX)

$$\left(\begin{bmatrix} \mathbf{l}_1^T \\ \mathbf{l}_2^T \end{bmatrix} \otimes \mathbf{X}^T \right) \mathbf{p} = \mathbf{0}$$

s.t. $\text{vec}(\mathbf{P}^T) = \underbrace{\begin{bmatrix} \mathbf{p}^1 \\ \mathbf{p}^2 \\ \mathbf{p}^3 \end{bmatrix}}_{12 \times 1}$, $\text{vec}(\mathbf{P}) = \underbrace{\begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \\ \mathbf{p}_4 \end{bmatrix}}_{12 \times 1}$

$$\underbrace{\left([\mathbf{x}_i]_\perp \otimes \mathbf{X}_i^T \right)}_{2 \times 12} \mathbf{p} = \mathbf{0} \quad \forall i \quad \mathbf{x}_i \longleftrightarrow \mathbf{X}_i$$

$$\underbrace{\begin{bmatrix} [\mathbf{x}_1]_\perp \otimes \mathbf{X}_i^T \\ [\mathbf{x}_2]_\perp \otimes \mathbf{X}_2^T \\ \vdots \\ [\mathbf{x}_n]_\perp \otimes \mathbf{X}_n^T \end{bmatrix}}_{2n \times 12} \underbrace{\mathbf{p}}_{12 \times 1} = \underbrace{\mathbf{0}}_{2n \times 1}$$

need
$\boxed{n \geq 6}$ (n is the number of point correspondences)
since 12 values in $\mathbf{p}$

$$\mathbf{A}\mathbf{p} = \mathbf{0}$$

solve for $\mathbf{p}$ (the null space of $\mathbf{A}$)

USE SVD TO COMPUTE THE NULL SPACE

note: it is faster to use QR or RQ decomposition
(this is what something like the HoloLens would use)

(FIND VECTOR CLOSEST TO NULL SPACE W/O BEING IN IT,
SINCE $\mathbf{A}$ IS FULL RANK AND ACTUAL SOLN IS JUST $\mathbf{0}$)

SVD Solution

$$\underbrace{\mathbf{A}}_{m \times q} = \underbrace{\mathbf{U}}_{m \times m} \underbrace{\boldsymbol{\Sigma}}_{m \times q} \underbrace{\mathbf{V}^T}_{q \times q} \qquad \underbrace{\mathbf{A}}_{2n \times 12} \underbrace{\mathbf{p}}_{12 \times 1} = \underbrace{\mathbf{0}}_{2n \times 1}$$

rank of A must be 11

FULL DECOMPOSITION

$$\mathbf{A} = \underbrace{[\mathbf{u}_1 | \mathbf{u}_2 | \ldots | \mathbf{u}_m]}_{2n \times 2n} \underbrace{\left[\begin{array}{ccc} \sigma_1 & & \mathbf{0} \\ & \sigma_2 \ddots & \\ \mathbf{0} & & \sigma_{\min(m,q)} \\ \hline & \mathbf{0} & \end{array} \right]}_{2n \times 12} \underbrace{\begin{bmatrix} \mathbf{v}^{1T} \\ \mathbf{v}^{2T} \\ \vdots \\ \mathbf{v}^{qT} \end{bmatrix}}_{12 \times 12}$$

$\mathbf{p}$ = right singular vector corresponding to smallest singular value
(last row of $\mathbf{V}^T$ / last column of $\mathbf{V}$)

* don't need to calculate $\mathbf{U}$

$\boxed{\mathbf{P} \text{ is just } \mathbf{p} \text{ reshaped into a } 3 \times 4 \text{ matrix}}$

DATA NORMALIZATION

a form of preconditioning [problem: measurements in pixel coordinates (e.g. 0-1920), but projective coordinates are 1 → vastly different magnitudes → numerical errors]

SOLUTION: move centroid to origin and rescale.

otherwise...