CSE 252A: Radiometry, Reflectance

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1 Radiometry

Let's define a local coordinate system at a point $\mathbf{P}$ on a surface. It will have a surface normal $\mathbf{N}$; there will be an angle $\theta \in [0, \frac{\pi}{2}]$ between each incident lighting direction and this normal. Among other things, θ determines the foreshortening of the surface: the smaller the angle, the more surface area there is for receiving light.

Often we'll have to integrate over a range of solid angles to determine how much light is striking a point. To integrate over solid angles, we need a notion of a **differential solid angle**:

$$d\omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2} = \sin\theta \, d\theta \, d\phi$$

With foreshortening, this becomes

$$d\omega = \frac{dA}{r^2} \cos\theta = \sin\theta \cos\theta \, d\theta \, d\phi$$

We usually integrate over θ and ϕ (rather than area, for example) because it's easy to sample and think about angles over a hemisphere.

- **Irradiance** $E(\mathbf{x})$ is the amount of light arriving at a point on a surface $\left(\frac{\text{watts}}{m^2}\right)$.
- **Radiance** $L(\mathbf{x}, \theta, \phi)$ is the amount of light passing through a point in some direction $\left(\frac{\text{watts}}{m^2 \text{sr}}\right)$.

$$L = \frac{\text{power}}{(dA \cos\alpha) \, d\omega}$$

Irradiance is power at a point; radiance is power at a point with a direction (e.g. on a ray).

Radiance coming in from solid angle $d\omega$ creates *irradiance*

$$L(\mathbf{x}, \theta, \phi) \cos\theta \, d\omega$$

due to foreshortening. So the total irradiance arriving at a point from all directions is

$$\int_{\text{hemisphere}} L(\mathbf{x}, \theta, \phi) \cos\theta \, d\omega$$

Our camera sensors produce outputs that are proportional to irradiance. Each pixel intensity equates to irradiance integrated over the pixel's area, a range of wavelengths, and for some period of time.

$$I = \int_t \int_\lambda \int_x \int_y E(x, y, \lambda, t) s(x, y) q(\lambda) dx \, dy \, d\lambda \, dt$$

for $s(x, y)$ the individual pixel response function, and $q(\lambda)$ the wavelength response function.

Sometimes a nonlinear response function is additionally applied to this:

$$I = R\left(\int_t \int_\lambda \int_x \int_y E(x, y, \lambda, t) s(x, y) q(\lambda) dx\, dy\, d\lambda\, dt\right)$$

for R determined by radiometric calibration.

2 Color Cameras

Cameras are meant to produce images to be seen by people. Since people see color according to S, M, L response curves (one for each type of cone), we have cameras filter light into R, G, B components.

We can do this, e.g., with a dichroic prism color camera that separates lights into three beams. This requires a lot of precision and is a high-end manufacturing choice.

Another approach is to apply a filter (e.g. a Bayer filter) directly to the sensor. Then we interpolate any values we don't have. Most commercial cameras do this. Note that a Bayer filter has more green than red or blue, since humans are more sensitive to green. The problem with the Bayer pattern is that it can introduce color artifacts, e.g. if the image is just black and white.

Yet another approach is to use a filter wheel. Here, we just keep switching out the filter in front of the lens (meaning we can have more than three filters). Of course, this is slow and can only be used for static scenes.

A newer approach, by Foveon, is the X3, which uses three stacked sensors (B, G, R). Light comes in through the top, hits blue, then gets absorbed and read out. Next it continues to pass through, hits green, and gets absorbed and read out. Finally it passes through once more, hits red, and gets absorbed and read out.

3 Light at Surfaces

Light interacts with different surfaces according to their different material properties. The amount of light reflected by a surface is described by the BRDF (ρ, f, or f_r), which says how much light from an incoming direction is reflected in an outgoing direction. The BRDF is the ratio of emitted radiance to incident irradiance:

$$\rho(\mathbf{x}; \theta_i, \phi_i; \theta_o, \phi_o) = \frac{L_o(\mathbf{x}; \theta_o, \phi_o)}{L_i(\mathbf{x}; \theta_i, \phi_i) \cos\theta_i\, d\omega}$$

It is used in the reflection equation as

$$L_r(\mathbf{x}, \omega_r) = \int_{H^2} f_r(\mathbf{x}, \omega_i \to \omega_r) L_i(\mathbf{x}, \omega_i) \cos\theta_i\, d\omega_i$$

where $L_r(\mathbf{x}, \omega_r)$ is the amount of light being reflected in a certain direction at a certain point. In the equation, we integrate all the light coming in from all directions which goes off in direction ω_r.

A common class of BRDFs is that of *isotropic BRDFs*, where if you rotate the surface around the normal and leave light source and viewing directions fixed, the BRDF value stays constant. This allows us to drop a parameter, making the BRDF a function of three variables instead of four. (Most materials are like this!)