CSE 152	Intro to Computer Vision	
Spring 2018	UC San Diego	Lecture 2

1 Image Formation

- In a camera, the process of image formation consists of (1) photons striking a grid of detector cells and (2) the charge at each detector cell (pixel) being read out as brightness. Typically, the detectors are CMOS or CCD, both of which convert photons into electric charge.
- Today's topic is *geometric* image formation, which means we'll concern ourselves with *where points in the world show up in the image* (i.e. geometry being that of the perspective projection model).
- In perspective projection, distant objects appear smaller than they would if they were closer.
  - A line from $(x, 0, z)$ to (x, y, z) projects to a line from $({}^{fx}/_{z}, 0)$ to $({}^{fx}/_{z}, {}^{fy}/_{z})$.
  - A line from $(x, 0, 2z)$ to $(x, y, 2z)$ projects to a line from $({}^{fx}/_{2z}, 0)$ to $({}^{fx}/_{2z}, {}^{fy}/_{2z})$.
- Geometric properties:
  - For the most part, lines are still lines after projection.
    - *Justification.* If the equation of a line is

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} m_x \\ m_y \end{bmatrix} p_z + \begin{bmatrix} b_x \\ b_y \end{bmatrix}$$

then each projected point is

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} fp_x/p_z \\ fp_y/p_z \end{bmatrix} = \begin{bmatrix} f(m_x p_z + b_x)/p_z \\ f(m_y p_z + b_y)/p_z \end{bmatrix}$$

and we have

$$p'_x = \frac{f(m_x p_z + b_x)}{p_z} \implies (p'_x - fm_x)p_z = fb_x$$
$$p_z = \frac{fb_x}{p'_x - fm_x}$$

which means

$$\begin{aligned}
p'_y &= \frac{f(m_y p_z + b_y)}{p_z} \\
p'_y p_z &= f(m_y p_z + b_y) \\
p'_y \frac{fb_x}{p'_x - fm_x} &= f\left(m_y \frac{fb_x}{p'_x - fm_x} + b_y\right) \\
p'_y &= \left(m_y \frac{fb_x}{p'_x - fm_x} + b_y\right)\left(\frac{p'_x - fm_x}{b_x}\right) \\
p'_y &= fm_y + \frac{b_y(p'_x - fm_x)}{b_x} \\
p'_y &= \frac{b_y}{b_x} p'_x + \left(fm_y - \frac{fb_y m_x}{b_x}\right) \quad \text{(the familiar } y = mx + b \text{ form)}
\end{aligned}$$

    - *Exception*: a line through the focal point $(0, 0, 0)$ projects to a point.
      - *Justification.* $(b_x, b_y) = (0, 0)$ and therefore $(p'_x, p'_y) = (fm_x, fm_y)$ every time.